# Lower hybrid drift wave in a plasma in the presence of dust particles of variable charge

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Lower hybrid waves in an inhomogeneous plasma with dust particles of variable charge have been theoretically estimated. An expression for growth rate has been obtained, which shows that the growth of such waves can be controlled by an appropriate selection of number of parameters, viz., the external magnetic field, the density inhomogeneity scale lengths, and charge on dust particles. [S1063-651X(97)07510-7]

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## I. INTRODUCTION

Dust particles can grow in plasmas during microelectronic fabrications or in surface processing reactors and remain electrically suspended there until they fall to a surface and contaminate it [1]. This "particle contamination" is a crucial concern in the manufacturing of microelectronic devices. Although modern clean rooms have reduced contamination, it is not generally recognized that the production tools and processes often contribute the major share of particle contamination. Present day plasmas are used extensively in semiconductor manufacturing for material etching, deposition, and surface treatment. Suspended particles have been observed at the plasma-sheath boundary in etching [2], deposition [3], and sputter [4] plasmas. When electrostatically isolated particles are exposed to a plasma, they rapidly develop a potential equal to the plasma floating potential, balancing the flux of positive and negative charges of the particle [5]. Since electrons are the primary carrier of mobile charges, the particles acquire a net negative charge, as confirmed experimentally [6]. These charged dust particles interact by means of their Coulomb repulsion and therefore form a strongly correlated system. Dusty plasmas, in contrast to ordinary plasmas, contain an additional species of large dust grains of radii in the range  $10^{-2} - 10^{-6}$  m. These particles are found to become negatively charged and the magnitude of this charge is of the order of  $10^3 e - 10^5 e$  for micrometer-sized dust particles [7]. The presence of these massive and highly charged particles can significantly change the collective behavior of the plasma in which they are suspended. It is also known that when dust particles are immersed in a partially or fully ionized plasma, the charge characteristics of dust particles makes them a special component of the plasma. One characteristic of dust particles is that they can be highly charged  $(Z_d = q_d/e)$  and another is that the charge on dust particles is not constant, i.e., it can fluctuate and also change if the dust density or plasma conditions change. Therefore, this process should significantly influence the collective properties of the plasma, where the Coulomb interactions play a dominant role.

Actually, plasmas with dust particles have become an important field of research after the encounters of Voyager with Jupiter, Saturn, and Uranus and of the Vega and Giotto probes with Halley's comet. Now we know that the planetary rings and cometary tails contain large dust components of micrometer and submicrometer sizes [8-10] and these par-

ticles can be charged by a combination of different processes such as solar ultraviolet radiation, plasma currents, and field emission. Dust particles in a plasma can charge up to various voltages with respect to the plasma depending on the ambient conditions [11,12]. Therefore, the presence of dust particles in a plasma modifies its collective properties. Dust particles are often negatively charged by plasma currents [13]. Dusty plasmas are basically characterized by electrons, ions, and dust particles. These dust particles are very small and are heavily charged due to various processes [14,15], viz., electron attachment, photoelectric emission, and secondary excitation. Therefore, the electric and magnetic forces on the dust grains plays a significant role on their motion. But researchers on dusty plasmas generally consider that the charge on dust grains is fixed, which is not true in reality, and the variation of charge on dust grains shows an alternative dimension to the research on dusty plasmas. Jana, Sen, and Kaw [16] have pointed out that during a collective oscillation, the charge on the grain is not fixed but fluctuates due to the oscillation of the self-consistent plasma currents flowing into the grains. It has also been experimentally observed that the charge fluctuation of dust particles can quite reasonably affect the plasma dynamics. Therefore, it is necessary to consider the dust charge fluctuation in a problem of dusty plasma to make it more viable for practical situations.

Most of the theoretical analyses of dusty plasmas assume that the charge on dust particles is constant. The variation of the dust charge had been pointed out by researchers several years ago [17,18]. However, the effect of dust charge variation on the study of waves and instabilities has begun only recently [16,19–21]. Very recently, the grain charge fluctuations have been verified experimentally [22–24] and also with the help of computer simulation [25,26].

Again, many studies have been made for the unmagnetized plasma, but only a few researchers have studied dusty plasmas in the presence of a magnetic field [27,28]. None of them have considered the charge variation on dust particles. But the time-dependent currents associated with the selfconsistent electric and magnetic fields of plasma modes flow onto the surface of the dust particle and therefore, the dust charge in the presence of plasma modes becomes a timedependent variable. Thus it is not surprising that astronomers and industrial researchers have investigated many physical processes [29–31] of dust particles.

The class of low-frequency electrostatic waves in magnetized plasma is fairly extensive. Among these low-frequency

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waves, drift waves merit special attention due to their role in enhanced (anomalous) plasma transport across magnetic fields. Drift waves were predicted by Rudakov and Sagdeev [32] and experimentally corroborated by D'Angelo and many other researchers [33]. Farengo, Gudzar, and Lee [34] have analytically investigated the lower hybrid drift (LHD) instability for a  $\theta$  pinch experiment. Recently, using a kinetic approach, Benkadda and Tsytovich [35] estimated the growth rate of a different type of drift instability, driven by the process of dust charging. They found that the growth rate is comparable to or much larger than the usual drift instability, even for a low dust density. We have obtained [36] similar results when a small percentage of heavy ion was present in a two-species plasma. In that paper [36] the energy calculation shows that the energy carried by the mode, due to the inclusion of a heavy ion, is very large.

The lower hybrid wave (LHW) is also useful since it is one of the best suited waves for the plasma heating. Since the frequency of this wave is not very high it is necessary to use special kinds of antennas and slow wave systems. As the inserted power level is rather high, nonlinear effects appearing at the plasma boundary become important. To give some insight into the way the LHW propagates one would expect the ions to oscillate at the plasma frequency when the electrons are completely immobilized by the magnetic field. Since, however, the electrons undergo a polarization drift, they act as a background dielectric medium with a dielectric constant  $\left[1 + (\omega_{pc}/\Omega_e)^2\right]$  to modify the frequency to  $(\omega_{\text{LH}})^2 = (\omega_{pi})^2 / [1 + (\omega_{pe} / \Omega_e)^2], \text{ where } \Omega_e \text{ is the electron}$ cyclotron frequency and  $\omega_{LH}$  is the lower hybrid frequency. If the wave vector has a small parallel component, the electrons oscillate along the magnetic field, leading to the following dispersion relation for the lower hybrid wave:  $\omega^2$  $=(\omega_{\text{LH}})^2 [1 + (M/m)(k_{\parallel}/k_{\perp})^2]$  [37]. The energy deposition via coupling of the lower hybrid wave into the plasma has been described clearly by Stix [38]. The lower hybrid wave has been extensively studied by many researchers [38–40].

The LHD instability is driven by cross-field currents in the presence of an inhomogeneity in the density. This instability has received considerable attention as a mechanism for anomalous transport during the post-implosion phases of rapidly pulsed  $\theta$  pinch experiments as well as a mechanism for producing flute perturbations during the implosion [34, 40]. Farengo, Guzdar, and Lee [34] have analytically investigated the effects of a finite parallel wave number and electron temperature gradients on the LHD instability in the parameter regime of a  $\theta$  pinch experiment [41]. In studying the LHD instability they did not consider the effect of dust particles. In addition, their study is based on kinetic theory, whereas the derivation in this paper is based on a fluid approach that is clear and concise. Since the plasmas in  $\theta$  pinches are collisionless with respect to the LHW, a collisionless treatment of the LHD wave has been done.

The following considerations have been made for the present problem of dusty plasma. (a) The charging of dust particles is due only to electron attachment and ion collection currents. The other charging processes, viz., photoemissions and secondary radiations, are negligible. (b) The dust particles are spherical in shape. (c) The radii of these dust particles are equal. This is a reasonable consideration when the radii of these grains lie in a narrow range. (d) The number

density of dust grains is not very high. The number density of electrons and ions is much larger than the number density of dust grains. (e) The frequency regimes considered here is  $\Omega_i, \Omega_d < \omega < \Omega_e; \quad k_Z V_{td} < k_Z V_{ti}, k_Z V_{te} < \omega;$  and  $\omega < k_\perp V_{ti}, k_\perp V_{td} < k_\perp V_{te}$ , where  $\Omega$  represents the gyrofrequency and the subscripts  $z, \perp$ , and t represent the z direction, the direction perpendicular to the magnetic field, and the temperature, respectively.

Using these assumptions, the dispersion relation for three different cases (due to variable charge on dust particles) has been obtained. The fluctuation of the charge on dust particles actually modifies the total current density of the plasma.

## **II. DISPERSION RELATION**

We have considered the frequency regime  $\Omega_i, \Omega_d < \omega$  $<\Omega_e; \quad k_Z V_{td} < k_Z V_{ti}, k_Z V_{te} < \omega; \quad \text{and} \quad \omega < k_\perp V_{ti}, k_\perp V_{td}$  $< k_{\perp} V_{te}$ . Here the drift frequency is larger than the ion cyclotron and dust cyclotron frequencies. Thus the magnetic effects on the ions and dust particles can easily be neglected. Hence the electrons of the system are considered to be magnetized, whereas the ions and dusts are unmagnetized. A homogeneous magnetic field  $(B_0)$  is applied in the z direction, while the inhomogeneity in the density occurs in the perpendicular direction (x). Here we have considered a flowing plasma and the zeroth-order bulk velocities  $V_{0i}$  are considered to be uniform. The bulk temperatures  $T_i$  are also uniform (where j=i, e, or d). If the zeroth-order number density  $n_{0i}$  occurs in the x direction, we do not find any contribution of  $V_{0i}$  in the x direction and this velocity term is then called the diamagnetic drift velocity, which is  $V_{0i}$  $=(k_bT_icL_i)/eB_0$ , where  $L_i=(1/n_{0i})(\delta n_{0i}/\delta n)$  is the density inhomogeneity scale length and a constant quantity along with  $B_0$ ,  $T_j$ , and  $V_{0j}$ . Again, since the perturbation in this system is isothermal and  $n_{0i}$  occurs in the x direction, the dispersion relation for a constant  $\omega$  and constant k can be found for a wave propagating in the perpendicular direction only; the excited wave in this case, a LHD wave, will also propagate in the direction perpendicular the applied magnetic field, i.e., in the y direction. With the help of continuity equations, equations of motion, and Poisson's equation for each species (i.e., for ions, electrons, and dusts) we obtain an equation for the density of electrons that satisfies the drift approximation; to give

$$\frac{\partial^2 n_e}{\partial t^2} + \frac{\partial}{\partial t} V_{0e} \nabla_{\perp} n_e - c n_{0e} \frac{\partial}{\partial t} \nabla \cdot \left( \frac{\nabla \phi \times \mathbf{z}}{B_0} \right) + e n_{0e} \nabla_z^2 \phi$$

$$+ \frac{e n_{0e}}{\Omega_e^2} \frac{\partial^2}{\partial t^2} \nabla_{\perp}^2 \phi - V_{te}^2 \nabla n_{n_e}^2 - \frac{c}{B_0} \frac{\partial}{\partial t} (\nabla \phi \times \mathbf{z}) \cdot \nabla n_{0e}$$

$$+ \frac{e}{m_e} \nabla_{\mathbf{z}} \phi \cdot \nabla n_{0e} + \frac{c}{B_0} \frac{\partial^2}{\partial t^2} \nabla_{\perp} \phi \cdot \nabla n_{0e}$$

$$- V_{te}^2 \frac{\nabla n_e \cdot \nabla n_{0e}}{n_{0e}} = 0,$$
(1)

where  $k_{\perp}^2 V_{te}^2 \ll \Omega_e^2$ ; in other words, if the thermal velocity exceeds the gyrofrequency of the electrons, the system will not be able to show any magnetic effect. After linearizing Eq. (1) we find

$$n_{e} = \frac{e n_{e0} \phi}{m_{e} k_{\perp}^{2} V_{te}^{2}} \left[ \frac{(\omega - k V_{0e})^{2}}{\Omega_{e}^{2}} k_{\perp}^{2} - \frac{k_{y} (\omega - k V_{0e})}{\Omega_{e} L_{e}} - k_{z}^{2} - \frac{i k_{y} (\omega - k V_{0e})^{2}}{\Omega_{e}^{2} L_{e}} \right],$$

$$(2)$$

where  $k_{\perp}^2 L_e \gg k_x$ ,  $V_{ij} = (k_B T_j / m_j)^{1/2}$ ,  $\Omega_j = (q_j B_0) / m_j c$ , and  $L_j = (1/n_{0j}) (\delta n_{0j} / \delta x)$  are the drift frequency, thermal velocities, gyrofrequencies, and density inhomogeneity scale lengths for each species.

Now we obtain the expression for  $n_i$  and  $n_d$  by Fourier transforming the continuity equations and the equations of motion for the ions and dust particles to get

$$n_{i} = \frac{ek_{\perp}^{2}n_{0i}\phi}{m_{i}(\omega - k_{\perp}V_{0i})^{2}} \left[1 - \frac{k_{\perp}^{2}V_{ti}^{2}}{(\omega - k_{\perp}V_{0i})^{2}}\right]^{-1}$$
(3)

and

$$n_d = \frac{n_{0d}}{m_d V_{td}^2} \left[ q_d \phi_0 + \phi q_{d0} \right].$$
(4)

Again, the dynamic equation connecting the self-consistent dust charge fluctuation is given by [16]

$$\frac{dq_d}{dt} + \eta q_d = |I_{e0}| \left[ \frac{n_i}{n_{0i}} - \frac{n_e}{n_{0e}} \right], \tag{5}$$

where  $\eta = [(e|I_{e0}|)/C][(1/T_e) + (1/\omega_0)], \quad \omega_0 = T_i - e\phi_{0f}, C(=a)$  is the grain capacitance,  $T_j$  is the temperature of the *j*th (*i*th or *e*th) particle,  $n_j$  is the perturbed density,  $n_{0j}$  is the equilibrium density,  $|I_{e0}|$  is the equilibrium electric current,  $\phi_{0f}$  is the equilibrium grain floating potential, and  $q_d$  is the dust charge fluctuation.

After linearizing Eq. (5) one can take three different conditions: (a) steady-state charge fluctuation, (b) spatially varying charge fluctuation, and (c) temporally varying charge fluctuation. For these three different cases three different perturbed dust densities have been obtained, respectively. With the help of equations for densities of each species, along with Poisson's equation, we get

$$\frac{m_i V_{ti}^2 (A+P) \omega^2 \left(k_{\perp}^2 + \frac{ik_y}{L_e}\right)}{m_e V_{te}^2 k_{\perp}^2 \Omega_e^2} - \frac{m_i V_{ti}^2 (A+P) \omega}{m_e V_{te}^2 k_{\perp}^2} \times \left[\frac{2kV_{0e}}{\Omega_e^2} \left(k_{\perp}^2 + \frac{ik_y}{L_e}\right) - \frac{k_y}{\Omega_e L_e}\right] - (A+P) \frac{m_i k_z^2 V_{ti}^2}{m_e k_{\perp}^2 V_{ti}^2} + \frac{m_i V_{ti}^2 (A+P) V_{0e}^2}{m_e V_{te}^2 \Omega_e^2} \left(k_{\perp}^2 + \frac{ik_y}{L_e}\right) - \frac{m_i V_{ti}^2 (A+P) k_y V_{0e}}{m_e V_{te}^2 k_{\perp} \Omega_e L_e}$$

$$-B - P + \frac{Z_d^2 \epsilon_{nd} m_i V_{ti}^2}{m_d V_{td}^2} = 0, \qquad (6)$$

where  $A = \varepsilon_{ne} - [(\varepsilon_{nd}|I_{e0}|)/e\eta], \quad P = [(Z_0\varepsilon_{nd}\phi_0|I_{e0}|]/(m_dV_{id}^2\eta)], \quad B = 1 - [\varepsilon_{nd}|I_{e0}|/e\eta], \quad \varepsilon_{ne} = n_{0e}/n_{0i}, \quad \varepsilon_{nd} = n_{0d}/n_{0i} \text{ and } Z_d = q_{0d}/e;$ 

$$(A_1 - iP_1) \frac{m_i V_{ti}^2 \left(k_{\perp}^2 + \frac{ik_y}{L_e}\right) \omega^2}{m_e V_{te}^2 \Omega_e^2} + \frac{m_i V_{ti}^2 (A_1 - iP_1) \omega}{m_e V_{te}^2}$$

$$\times \left\{ \frac{2k_{\perp}V_{0e} \left(k_{\perp}^{2} + \frac{ik_{y}}{L_{e}}\right)}{\Omega_{e}^{2}} - \frac{k_{y}}{\Omega_{e}L_{e}} \right\} + B_{1} - iP_{1} + \frac{m_{i}V_{ti}^{2}}{m_{e}V_{te}^{2}}$$
$$\times (A_{1} - iP_{1}) \left\{ k_{z}^{2} - k_{\perp}^{2}V_{0e}^{2} \left(\frac{k_{\perp}^{2} + \frac{ik_{y}}{L_{e}}}{\Omega_{e}^{2}} + \frac{k_{\perp}k_{y}V_{0e}}{\Omega_{e}L_{e}}\right)$$
$$+ \frac{m_{i}Z_{d}^{2}\varepsilon_{nd}V_{ti}^{2}}{m_{e}V_{te}^{2}} = 0,$$
(7)

where  $A_1 = \varepsilon_{ne} + [(i\varepsilon_{nd}|I_{e0}|)/ek_{\perp}V_{0d}], \quad B_1 = 1 + [(i\varepsilon_{nd}|I_{e0}|)/ek_{\perp}V_{0d}], \quad \text{and} \quad P_1 = [(Z_d\varepsilon_{nd}\phi_0|I_{e0}|)/k_{\perp}m_dV_{0d}V_{td}^2];$  and

$$\begin{split} \frac{m_i V_{ti}^2 A_2 \omega^3 \left(k_{\perp}^2 + \frac{ik_y}{L_e}\right)}{m_e V_{te}^2 k_{\perp}^2 \Omega_e^2} &- \frac{m_i V_{ti}^2 \omega^2}{m_e V_{te}^2 k_{\perp}^2} \\ \times \left[ (2A_2 k_{\perp} V_{0e} - iP_2) \frac{k_{\perp}^2 + \frac{ik_y}{L_e}}{\Omega_e^2} - \frac{k_y A_2}{\Omega_e L_e} \right] \\ &- \omega \left[ \frac{m_i V_{ti}^2 A_2}{m_e V_{te}^2 k_{\perp}^2} \left\{ k_z^2 - k_{\perp}^2 V_{0e}^2 \frac{\left(k_{\perp}^2 + \frac{ik_y}{L_e}\right)}{\Omega_e^2} + \frac{k_{\perp} k_y V_{0e}}{\Omega_e L_e} \right\} \right. \\ &+ \frac{iP_2 m_i V_{ti}^2}{k_{\perp}^2 m_e V_{te}^2} \left\{ 2k_{\perp} V_{0e} \frac{\left(k_{\perp}^2 + \frac{ik_y}{L_e}\right)}{\Omega_e^2} - \frac{k_y}{\Omega_e L_e} \right\} \\ &+ B_2 - \frac{Z_d^2 \varepsilon_{nd} m_i V_{ti}^2}{m_d V_{td}^2} \right] - \frac{im_i V_{ti}^2 P_2}{k_{\perp}^2 m_e V_{te}^2} \\ &\times \left[ k_z^2 - k_{\perp}^2 V_{0e}^2 \frac{k_{\perp}^2 + \frac{ik_y}{L_e}}{\Omega_e^2} + \frac{k_{\perp} k_y V_{0e}}{\Omega_e L_e} \right] - iP_2 = 0, \end{split}$$

(8)

where  $A_2 = \varepsilon_{ne} - [(i\varepsilon_{ne}|I_{e0}|)/e\omega], B_2 = 1 - [(i\varepsilon_{nd})|I_{e0}|/e\omega]],$ and  $P_2 = [(Z_d\varepsilon_{nd}\phi_0|I_{e0}|)/m_dV_{td}^2].$ 

These are three dispersion relations for three different conditions considered here. For a uniform, quiet, and dust-free plasma, the dispersion relation reduces to  $\omega^2 \equiv \omega_{pi}[I + (k_z^2 m_i/k_\perp^2 m_e) + k_\perp^2 \lambda_i^2 \{I + (\omega_{pe}/\Omega_e)^2\}]$  where  $\lambda_i$  is the ion Debye length. At  $k_\perp^2 \lambda_i^2 [I + (\omega_{pe}/\Omega_e)^2] \leq 1$ , we obtain the

linear lower hybrid wave (as given in [37]) from the first root.

### **III. GROWTH RATE**

The growth rate of the excited lower hybrid drift wave in a plasma with variable dust grains can be estimated for three different cases from Eqs. (9)-(11) as

$$\gamma = \frac{k_{\perp}^{2}k_{y}\Omega_{i}\Omega_{e}\left[\frac{(B+P)V_{te}^{2}}{(A+P)V_{ti}^{2}} + \frac{m_{i}}{m_{e}}\frac{k_{z}^{2}}{k_{\perp}^{2}} + \frac{k_{y}V_{0e}}{k_{\perp}\Omega_{e}L_{e}} - \frac{Z_{d}^{2}\varepsilon_{nd}m_{e}V_{te}^{2}}{(A+P)m_{d}V_{id}^{2}}\right] / 2L_{e}\left(k_{\perp}^{4} + \frac{k_{y}^{2}}{L_{e}^{2}}\right) \\ = \frac{k_{\perp}^{4}\Omega_{e}^{2}}{\left[\frac{k_{\perp}^{4}\Omega_{e}^{2}}{k_{\perp}^{4} + \frac{k_{y}^{2}}{L_{e}^{2}}}\left\{\frac{B+P}{A+P}\frac{m_{e}V_{te}^{2}}{m_{i}V_{ti}^{2}} + \frac{k_{z}^{2}}{k_{\perp}^{2}} + \frac{k_{y}V_{0e}}{k_{\perp}L_{e}\Omega_{e}} - \frac{Z_{d}^{2}\varepsilon_{nd}m_{e}V_{te}^{2}}{(A+P)m_{d}V_{id}^{2}}\right] / 2L_{e}\left(k_{\perp}^{4} + \frac{k_{y}^{2}}{L_{e}^{2}}\right), \quad (9)$$

$$\frac{\gamma_{i}\Omega_{e}}{\left[\frac{k_{\perp}^{4}\Omega_{e}^{2}}{k_{\perp}^{4} + \frac{k_{y}^{2}}{L_{e}^{2}}}\left[m_{i}k_{y}\left(\frac{k_{z}^{2} + \frac{k_{\perp}k_{y}V_{0e}}{m_{e}L_{e}}\right) + \frac{V_{te}^{2}\Omega_{e}^{2}}{k_{\perp}^{2} + P_{1}^{2}}\left[\frac{P_{1}}{k_{\perp}L_{e}\Omega_{e}} - \frac{Z_{d}^{2}\varepsilon_{nd}m_{e}V_{te}^{2}}{(A+P)m_{d}V_{td}^{2}}\right] - k_{\perp}^{2}V_{0e}^{2}\right]^{1/2}}, \quad (9)$$

$$\frac{\gamma_{i} = \frac{\Omega_{i}\Omega_{e}}{2\left(k_{\perp}^{4} + \frac{k_{y}^{2}}{L_{e}^{2}}\right)}\left[m_{i}k_{y}\left(\frac{k_{z}^{2} + \frac{k_{\perp}k_{y}V_{0e}}{m_{e}L_{e}}\right) + \frac{V_{te}^{2}\Omega_{e}^{2}}{k_{\perp}^{2} + P_{1}^{2}}\left[\frac{P_{1}}{V_{ii}^{2}}\left(A_{1}k_{\perp}^{2} + \frac{P_{1}k_{y}}{L_{e}}\right) - \left(P_{1}k_{\perp}^{2} - \frac{A_{1}k_{y}}{L_{e}}\right)\left(\frac{m_{i}Z_{d}^{2}\varepsilon_{nd}}}{m_{d}V_{id}^{2}} + \frac{B_{1}}{V_{ii}^{2}}\right)\right]^{1/2}}{\left[\frac{V_{ie}^{2}\Omega_{e}k_{y}P_{1}A_{1}\Omega_{i}}{V_{ie}^{2}}\left(\frac{k_{\perp}^{2} + \frac{k_{\perp}k_{y}V_{0e}}}{k_{\perp}^{4} + \frac{k_{y}^{2}}{L_{e}^{2}}}\right) + \frac{K_{i}^{2}}{R_{\perp}^{2} + \frac{K_{i}^{2}}{R_{\perp}^{2}}}\left(\frac{k_{\perp}^{2}k_{\perp}^{2}}{m_{i}^{2}}\left(\frac{A_{1}k_{\perp}^{2}Z_{d}^{2}\varepsilon_{nd}}}{m_{i}^{2}V_{id}^{2}} + \frac{B_{1}P_{1}k_{\perp}^{2}}{m_{i}^{2}V_{id}^{2}} + \frac{B_{1}P_{1}k_{\perp}^{2}}{m_{i}^{2}V_{id}^{2}} + \frac{B_{1}P_{1}k_{\perp}^{2}}{m_{i}^{2}V_{id}^{2}}\right)\right]^{1/2}, \quad (10)$$

and

$$\gamma_{2} = \frac{\frac{\Omega_{e}\Omega_{i}}{2\left(k_{\perp}^{4} + \frac{k_{y}^{2}}{L_{e}^{2}}\right)} \left[\frac{k_{y}k_{z}^{2}m_{i}}{m_{e}k_{\perp}L_{e}} + \frac{k_{y}^{2}V_{0e}}{\Omega_{i}L_{e}^{2}} + \frac{V_{te}^{2}k_{\perp}k_{y}}{A_{2}\Omega_{i}L_{e}} - \frac{k_{\perp}k_{y}Z_{d}^{2}\varepsilon_{nd}m_{i}}{m_{e}A_{2}L_{e}}\right] + \frac{P_{2}V_{0e}}{A_{2}}}{\left[\frac{\Omega_{e}^{2}}{\left(k_{\perp}^{4} + \frac{k_{y}^{2}}{L_{e}^{2}}\right)} \left\{\frac{m_{e}V_{te}^{2}k_{\perp}^{2}}{m_{i}V_{ti}^{2}A_{2}} - \frac{Z_{d}^{2}\varepsilon_{nd}k_{\perp}^{2}}{A_{2}} + k_{z}^{2} + \frac{k_{\perp}k_{y}V_{0e}}{\Omega_{e}L_{e}} - \frac{k_{\perp}^{2}}{k_{\perp}^{2}}L_{e}^{2}A_{2}\Omega_{e}}\right\}}\right]^{1/2}}.$$
(11)

From these equations it can be inferred that as the charge on dust particles increases, the growth rate, in all cases, for the lower hybrid drift wave will decrease. Therefore, the quantity  $Z_d$  is one of the controlling parameter of the system. The electron density inhomogeneity scale length also affects the system in such a manner that its increment will decrease the growth of the excited wave. The number density is also a crucial factor that is related to this growth of the lower hybrid drift wave in the presence of charged dust particles in a complicated way. The streaming velocity of electrons will directly enhance the growth of the excited wave. The number density of each species is responsible for the variation in the growth rate of the excited wave. The external magnetic field directly affects the growth of the excited wave in such a manner that its increment will enhance the growth rate of the LHD wave as the gyration of each particle thereby increases. Therefore, by taking a strict measure towards these parameters, one can easily control the excitation of the LHD wave in a plasma with dust particles of variable charges.

Thus it can be inferred from the present work that this type of drift wave inside a plasma can be stabilized with the help of generating dust particles inside the system, which generally happens in all plasma machines, and as the charge on dust grains increases the growth of the excited wave will decrease. In this problem, the variation of the charge on grains is due only to electron attachment processes and the effects due to secondary radiations or photoelectric emissions have not been taken into account.

### IV. RESULTS AND DISCUSSION

The excitation of lower hybrid drift waves has been derived in a plasma with dust grains of variable charges. These charged dust particles are generally available in all plasma systems. Following a general formalism that involves solving hydrodynamic and electromagnetic equations, the expression for the dispersion relation of the LHD wave has been obtained. It is to be emphasized here that the dust grains are massive charged particles inside a plasma system, so since the system is a collection of charged particles, it will react with the dust particles to restructure its dynamical behavior. Also, since these dust grains are very massive, the low-frequency instabilities in the system will be encouraged due to their presence. In this problem, the charging of dust grains has been considered only through their interaction with the plasma current. Other processes for the charging of the dust grains, such as secondary radiation or photoelectric emission have not been taken into account. The estimation for the excitation of the LHD instability, in this paper, has accounted for the variable charge on dust grains since most of the current observations show that the dust particles inside a plasma system have a fluctuating charge.

Looking at the expressions for the growth rate and frequency of the LHD wave, the following general observations can be made. (i) The number density of each species directly affects this growth instability. (ii) The streaming velocity of each particle affects the system. Since the streaming velocity of electrons is generally quite high due to the smaller inertia, electrons participate in the process quite easily. (iii) The external magnetic field also plays a vital role. For higher values of the magnetic field, particles gyrate more and since electrons are the lightest particle, higher values of the magnetic field raise the gyration frequency of electrons

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so as to get more growth of excited waves. (iv) The thermal energy of the system is also responsible for an increment in the growth rate of the LHD wave. (v) The charge on dust grains affect the growth as well as the frequency of the excited wave.

Finally, all three cases considered here describe the fact that the excitation of the lower hybrid drift wave in a plasma with a charged dust population (where the charge on those dust particles varies with time or space) is dependent on the number of quantities. Also, points (iv) and (v) stated above are interdependent. Therefore, while controlling LHD waves one should be very careful about the interdependent components.

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